Level5opaedia

'A level is a level'

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Numbers and the Number System

Use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 and 1000 and explain the effect

Know that, e.g.
- in 5.239 the digit 9 represents nine thousandths, which is written as 0.009
- the number 5.239 in words is 'five point two three nine' not 'five point two hundred and thirty five'
- the fraction 5 and 239/1000 is read as 'five and two hundred and thirty-nine thousandths

Complete statements such as, e.g.
- 4 ÷ 10 = ⁊
- 4 ÷ ⁊ = 0.04
- 0.4 x 10 = ⁊
- 0.4 x ⁊ = 400
- 0.4 ÷ 10 = ⁊
- 0.4 ÷ ⁊ = 0.004
- ⁊ ÷ 100 = 0.04

How would you explain that 0.35 is greater than 0.035?

My calculator display shows 0.001. Tell me what will happen when I multiply by 100. What will the display show?

I divide a number by 10, and then again by 10. The answer is 0.3. What number did I start with? How do you know?

How would you explain how to multiply a decimal by 10 …., how to divide a decimal by 100? ….

Round decimals to the nearest decimal place and order negative numbers in context

Round, e.g.
- 2.75037 to 1 decimal place
- 176.05 to 1 decimal place
- 24.9316 to 2 decimal places
- 137.4996 to 3 decimal places

Order the following places from coldest to warmest:
- Moscow, Russia: 4°C
- Oymyakou, Russia: -96°C
- Vostok, Antarctica: -129°C
- Rogers Pass, Montana, USA: -70°C
- Fort Selkirk, Yukon, Canada: -74°C
- Northice, Greenland: -87°C
- Reykjavik, Iceland: 5°C

Show me:
- a number that rounds to 4.3 to 1 decimal place
- a situation where you would need to order negative numbers

What is wrong:
- 2.399 rounds to 2.310 to 2 decimal places
- -6 is smaller than -4

What is the same / different:
- 72.344 and 72.346

True / Never / Sometimes:
- 3.5 is closer to 4 than it is to 5
- -36 is bigger than -34

Convince me that:
- 8.4999 rounds to 8.5 to 3 decimal places
- -14 is greater than -16

Recognise and use number patterns and relationships

Continue sequences involving decimals

Describe number relationships including multiple, factor and square

Understand and use prime numbers

Show me:
- A prime number greater than 100
- Two prime numbers that add to 98

True/Never/Sometimes: The sum of the factors of a number (except the number itself) have a total greater than that number

What is the same different about
- 4.3, 4.6, 4.9, 5.2, …
- 16.8, 17.1, 17.4, 17.7, …
- 9.4, 9.1, 8.8, 8.5, …

Convince me that 1 is not a prime number

Use equivalence between fractions and order fractions and decimals

Find two fractions equivalent to 4/5	Give me two equivalent fractions. How do you know they are equivalent?
Show that 12/18 is equivalent to 6/9, 4/6 or 2/3	Give me some fractions that are equivalent to … How did you do it?
Find the unknown numerator or denominator in equivalent fraction statements	What sets of equivalent percentages, fractions and decimals do you know? Explain how you can use these to find other equivalent sets.
Order fractions with different denominators	
Know there is more than one way to find a percentage using a calculator. For example, to find 12% of 45: Convert a percentage calculation to an equivalent decimal calculation 0.12 x 45. Or, convert a percentage calculation to an equivalent fraction calculation 12/100 x 45. Recognise that the second method is less efficient than the first.	How would you use a calculator to find 12% of … using: (i) the x and ÷ keys (ii) only the x key? Why do both work? Which method is more efficient?
	10% is the same as 1/10 so 20% must be the same as 1/20. Is this true or false? Why?
Convert fractions such as 2/5 into tenths or hundredths and express them as decimals or percentages and vice versa	What percentages of given quantities can you easily work out in your head? Talk me through a couple of examples.
	When calculating percentages of quantities, what percentages do you usually start from? How do you use this percentage to work out others?

Reduce a fraction to its simplest form by cancelling common factors

Cancel these fractions to their simplest form by looking for highest common factors: • 9/15 • 12/18 • 42/56	What sets of equivalent percentages, fractions and decimals do you know? Explain how you can use these to find other equivalent sets.
	How would you use a calculator to find 12% of … using: (i) the x and ÷ keys (ii) only the x key? Why do both work? Which method is more efficient?
	10% is the same as 1/10 so 20% must be the same as 1/20. Is this true or false? Why?
	What percentages of given quantities can you easily work out in your head? Talk me through a couple of examples.
	When calculating percentages of quantities, what percentages do you usually start from? How do you use this percentage to work out others?

Understand simple ratio

Write 16:12 in its simplest form	Show me a ratio which simplifies to 2:7
A teaspoon holds 5ml of medicine and a bottle holds 100ml of medicine. Find the ratio of the capacity of the teaspoon to the capacity of the bottle. Write the answer in its simplest form	What is wrong: To simplify the ratio 32:48 keep dividing both sides by 2 until you can't do it any more
Understand the meaning of 'mix sand and cement in the ratio 5:1'	What is the same / different: 4:5 and £4:500p 2:3, 34:51 and 3:2
	Convince me that 19:95 ≡ 1:5

Calculating

Use known facts, place value, knowledge of operations and brackets to calculate including using all four operations with decimals to two places

Multiply or divide decimal numbers by a single digit e.g.
- 31.62 × 7
- 109.6 ÷ 8
- 239.22 ÷ 6

Know and use the order of operations, including brackets

Use factors, e.g.
- 3.2 x 30: 3.2 x 10 = 32: 32 x 3 = 96
- 156 ÷ 6: 156 ÷ 3 = 52   52 ÷ 2 = 26

Use partitioning, e.g. for multiplication, partition either part of the product: 7.3 x 11 = (7.3 x 10) + 7.3 = 80.3

Use 1/5 = 0.2 to convert fractions to decimals mentally. e.g. 3/5 = 0.2 x 3 = 0.6

Explain how would you do this multiplication by using factors, e.g. 5.8 x 40

What clues do you look for when deciding if you can do a multiplication mentally? e.g. 5.8 x 40

Give an example of how you could use partitioning to multiply a decimal by a two digit whole number, e.g. 5.3 x 23.

Use a calculator where appropriate to calculate fractions/percentages of quantities/measurements

Use mental calculations, e.g.
- 1/8 of 20 = 2.5; find one quarter and halve it
- 75% of 24 = 18; find 50% then 25% and add the results
- 15% of 40 = 6; find 10% then 5% and add the results
- 40% of 400kg = 160kg; find 10% then multiply by 4

Calculate simple fractions or percentages of a number/quantity e.g. ⅜ of 400g or 20% of £300

What percentages of given quantities can you easily work out in your head? Talk me through a couple of examples.

When calculating percentages of quantities, what percentages do you usually start from? How do you use this percentage to work out others?

Understand and use an appropriate non-calculator method for solving problems that involve multiplying and dividing any three digit number by any two-digit number

Understand and use an appropriate non-calculator method for solving problems that involve multiplying and dividing any three-digit number by any two-digit number, e.g. 6.24 × 8, 673 × 24, 3199 ÷ 7

Give pupils some examples of multiplications and divisions with mistakes in them. Ask them to identify the mistakes and talk through what is wrong and how they should be corrected.

Solve simple problems involving ordering, adding, subtracting negative numbers in context

Work out the resulting temperature after a change which passes 0°C, or one which involves negative numbers only

Solve problems involving overdrawn amounts on bank statements

'Addition makes numbers bigger.' When is this statement true and when is it false?'

Subtraction makes numbers smaller.' When is this statement true and when is it false?

The answer is -7. Can you make up some addition/ subtraction and calculations with the same answer.

The answer on your calculator is -144. What keys could you have pressed to get this answer?

Solve simple problems involving ratio and direct proportion

Solve simple problems involving ratio and direct proportion, beginning to use multiplication rather than trial and improvement to solve ratio problems	The ratio of boys to girls in a school is 4:5. How many pupils could be in the school? What is wrong: A map is drawn to the scale 1:500. Therefore 1cm on the map represents 500m on the ground. A number of cubes are arranged and the ratio of red cubes to green cubes is 2:7. If the pattern is continued until there are 28 cubes of one colour, how many cubes of the other colour will there be?

Apply inverse operations and approximate to check answers to problems are of the correct magnitude

Discuss questions such as, e.g. • A girl worked out the cost of 8 bags of apples at 47p a bag. Her answer was £4.06. Without working out the answer, say whether you think it is right or wrong. • I buy six items costing 76p, 89p, 36p, £1.03, 49p and 97p. I give the shop assistant a £10 note and get £3.46 change. I immediately think the change is wrong. Without calculating the sum, explain why you think I am right. • A boy worked out £2.38 + 76p on a calculator. The display showed 78.38. Why did the calculator give the wrong answer? Check by doing the inverse operation, e.g. • use a calculator to check 43.2 x 26.5 = 1144.8 with 1144.8 ÷ 43.2 • 3/5 of 320 = 192 with 192 x 5 ÷ 3 • 3 / 7 = 0.4285714… with 7 x 0.4285714	Roughly what answer do you expect to get? How did you come to that estimate? Do you think your estimate is higher or lower than the real answer? Explain why. How could you use inverse operations to check that a calculation is correct?

Algebra

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<td colspan="2">Construct, express in symbolic form, and use simple formulae involving one or two operations</td>
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Substitute integers into simple formulae

Simplify P=x+x+y+y

Write P = 2(x+y) as P=2x+2y

Recognise that in the expression 2 + 5a the multiplication is to be performed first

Understand that the letter stands for an unknown number or variable number and not a label, e.g. '5a' cannot mean '5 apples'

Understand the difference between expressions such as:
- 2n and n+2
- 3(c + 5) and 3c + 5
- n² and 2n
- 2n² and (2n)²

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When you substitute a = 2 and b = 7 into the formula t = ab + 2a you get 18. Can you make up some more formulae that also give t = 18 when a = 2 and b = 7 are substituted?

Give pupils examples of multiplying out a bracket with errors. Ask them to identify and talk through the errors and how they should be corrected, e.g.
- 4(b+2) = 4b + 2
- 3(p -4) = 3p - 7
- -2 (5 - b) = ¯10 -2b
- 12 – (n – 3) = 9 – n

Similarly for simplifying an expression.

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<tr>
<td colspan="2">Use and interpret coordinates in all four quadrants</td>
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Plot and interpret graphs such as
- y = x
- y = 2x
- y = x + 1
- y = x - 1

Given the coordinates of three vertices of a parallelogram, find the fourth

You might like to try:
x is a cross, wise up!

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Why is the point (3, 6) not on the line y = x + 2 ?

What are the important conventions when describing a point using a coordinate?

I'm thinking of a co-ordinate that I want you to plot. I can only answer 'yes' and 'no'. Ask me some questions to find out the co-ordinate so you can plot it.

How do you use the scale on the axes to help you to read a co-ordinate that has been plotted?

How do you use the scale on the axes to help you to plot a co-ordinate accurately?

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Shape, Space and Measures

Use a wider range of properties of 2-D and 3-D shapes and identify all the symmetries of 2-D shapes	
Find lines of reflection symmetry in shapes and diagrams Draw shapes with a fixed number of lines of symmetry Recognise the rotation symmetry of familiar shapes, such as parallelograms and regular polygons. Also: reason about shapes, positions and movements, e.g. ▪ visualise a 3-D shape from its net and match vertices that will be joined ▪ visualise where patterns drawn on a 3-D shape will occur on its net	Sketch me a quadrilateral that has one line of symmetry, two lines, three lines, no lines etc. Can you give me any others? What is the order of rotational symmetry of each of the quadrilaterals you sketched?

Use language associated with angle and know and use the angle sum of a triangle and that of angles at a point	
Calculate 'missing angles' in triangles, including isosceles triangles or right angled triangles, when only one/one other angle is given Calculate angles on a straight line or at a point such as the angle between the hands of a clock, or intersecting diagonals at the centre of a regular hexagon Understand 'parallel' and begin to understand 'perpendicular' in relation to edges or faces Classify quadrilaterals, including trapezium, using properties such as number of parallel sides	Is it possible to draw a triangle with: i) one acute angle ii) two acute angles iii) one obtuse angle iv) two obtuse angles Give an example of the three angles if it is possible, explain why if it is impossible.

Reason about position and movement and transform shapes	
Reflect shapes in oblique (45°) mirror lines where the shape either does not touch the mirror line, or where the shape crosses the mirror line Reflect shapes not presented on grids, by measuring perpendicular distances to/from the mirror Reflect shapes in two mirror lines, where the shape is not parallel or perpendicular to either mirror Rotate shapes, through 90° or 180°, when the centre of rotation is a vertex of the shape and recognise such rotations Translate shapes along an oblique line	Make up a reflection/rotation that is easy to do. Make up a reflection/rotation that is hard to do. What makes it hard? What clues do you look for when deciding whether a shape has been reflected or rotated?

Measure and draw angles to the nearest degree, when constructing models and drawing or using shapes	
Measure and draw reflex angles to the nearest degree, when neither edge is horizontal / vertical Construct a triangle given the length of two sides and the angle between them (accurate to 1mm and 2°)	Why is it important to estimate the size of an angle before measuring it? What important tips would you give to a person about using a protractor? How would you draw a reflex angle with a 180° protractor?

Read and interpret scales on a range of measuring instruments, explaining what each labelled division represents	
	When reading scales how do you decide what each division on the scale represents?

Solve problems involving the conversion of units and make sensible estimates of a range of measures in relation to everyday situations

Change a larger unit into a smaller one. e.g.
- Change 36 centilitres into millilitres
- Change 0.89km into metres
- Change 0.56 litres into millilitres

Change a smaller unit into a larger one. e.g.
- Change 750 g into kilograms
- Change 237 ml into litres
- Change 3 cm into metres
- Change 4mm into centimetres

Work out approximately how many km are equivalent to 20 miles

Solve problems such as 1.5kg ÷ 30g

Explain what each labelled division represents on a scale

What rough metric equivalents of imperial measurements do you know?

How would you change metres into feet, km into miles etc?

£4 for a gallon of petrol or 80p a litre. Which is the best value for money? Explain.

Which is bigger 200cm or 20 000mm? Explain how you worked it out.

Give me another measurement that is the same as 3m.

What clues do you look for when deciding which metric unit is bigger?

Explain how you convert metres to centimetres.

How do you change g into kilograms, ml into litres, km into metres etc?

Understand and use the formula for the area of a rectangle and distinguish area from perimeter

Find the length of a rectangle given its perimeter and width

Find the area or perimeter of simple L shapes, given some edge lengths

Draw a parallelogram or trapezium of a given area on a square grid

Reason about special triangles and quadrilaterals e.g. given the perimeter and one side of an isosceles triangle, find both possible triangles

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For a given area (e.g. $24cm^2$) find as many possible rectangles with whole number dimensions as you can. How did you do it?

For compound shapes formed from rectangles: How do you go about finding the dimensions needed to calculate the area of this shape? Are there other ways to do it?

Handling Data

Ask questions, plan how to answer them and collect the data required

Plan to answer questions such as:
- Which football team has the best goal-scoring record?
- What method of travel to school has the shortest journey time?
- Which newspaper is the easiest to read?

What was important in the way that you chose to collect data?

Why did you choose to ask that many people / select that amount of data?

How will you make sense of the data you have collected? What options do you have in organising the data?

How will you make use of the data you have collected?

In probability, select methods based on equally likely outcomes and experimental evidence, as appropriate

Describe and predict outcomes from data using the language of chance or likelihood

Compare two spinners to find which is more likely to result in an even number

Decide if a probability can be calculated or if it can only be estimated from the results of an experiment

On a fair die what is the probability of rolling, e.g.
- 5?
- an odd number?
- 0?
- a number greater than 2?
- a prime number?
- a number lying between 0 and 7?

Make up examples of equally likely outcomes with given probabilities e.g. 0.5, 1/6, 0.2 etc. Justify your answers.

'When you spin a coin, the probability of getting a head is 0.5. So if you spin a coin ten times you would get exactly 5 heads'. Is this statement true or false? Why?

Understand and use the probability scale from 0 to 1

On a fair die what is the probability of rolling, e.g.
- 5?
- an odd number?
- 0?
- a number greater than 2?
- a prime number?
- a number lying between 0 and 7?

Mark these probabilities on a probability scale

Show me an event which should be placed at ½, ¼, … on the probability scale

What is wrong:
- with any probability greater than 1
- with any probability less than 0

What is the same / different with a probability scale marked with:
- fractions
- decimals
- percentages
- and words?

Understand and use the mean of discrete data and compare two simple distributions, using the range and one of mode, median or mean

Describe and compare two sets of football results, by using the range and mode	Find 5 numbers that have a mean of 6 and a range of 8. How did you do it? What if the median was 6 and the range 8? What if the mode was 6 and the range 8?
Solve problems such as, 'Find 5 numbers where the mode is 6 and the range is 8'	
Use the mean of a set of measurements from a science experiment	Two distributions both have the same range but the first one has a median of 6 and the second has a mode of 6. Explain how these two distributions may differ.
How do pupils travel to school? Compare the median and range of the times taken to travel to school for two groups of pupils such as those who travel by bus and those who travel by car.	
Which newspaper is easiest to read? In a newspaper survey of the numbers of letters in 100-word samples, compare the mean and the range ▪ Tabloid: mean 4.3 and range 10, ▪ Broadsheet: mean 4.4 and range 14	

Understand that different outcomes may result from repeating an experiment

Carry out a coin-tossing / toast-dropping / peanut-burning experiment and compare results with others, appreciating why the results are variable	You toss a coin 100 times and count the number of times you get a head. A robot is programmed to toss a coin 1000 times. Who is most likely to be closer to getting an equal number of heads and tails? Why?

Interpret graphs and diagrams, including pie charts, and draw conclusions

See the full range of examples on pages 268 – 270 of the KS3 Framework supplement of examples	Make up a statement or question for this chart/graph using one or more of the following key words: ▪ total, range, mode; ▪ fraction, percentage, proportion.
Complete a 2-way table, given some of the data	
Interpret bar graphs with grouped data	
Interpret and compare pie charts where it is not necessary to measure angles	
Read between labelled divisions on a scale, for example read 34 on a scale labelled in tens or 3.7 on a scale labelled in ones, and find differences to answer, 'How much more…?'	
Recognise the difference between discrete and continuous data	
Recognise when information is presented in a misleading way, for example compare two pie charts where the sample sizes are different	
When drawing conclusions, identify further questions to ask	

Create and interpret line graphs where the intermediate values have meaning

Draw and use a conversion graph for pounds and Euros	Show me and example of a line graph where the intermediate values do not have a meaning
	What is wrong with this graph? (a line graph where the intermediate values do not have a meaning)
	Convince me that you can use this graph (conversion graph between litres and gallons – up as far as 20 gallons) to find out how many litres are roughly equivalent to 75 gallons